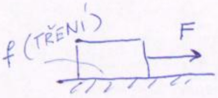
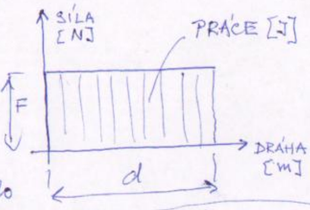
Ten, kdo nemá zápočet z PP1 nebude zkoušen z PP2
zápočty se uzn. ještě v dalším 2r. obd.

1

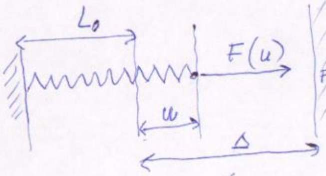
Práce vnějších sil

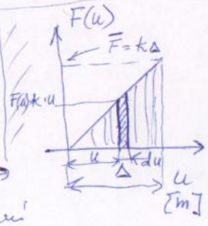




$W = F \cdot d$

Disipace → teplo

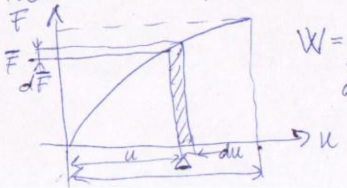




$F(u) = k \cdot u$ (lineární záv.)

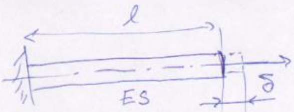
$$W = \int_0^{\Delta} F(u)\,du = \int_0^{\Delta} k \cdot u\,du = k \cdot \frac{\Delta^2}{2} = \frac{1}{2} F \cdot \Delta$$

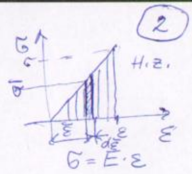
Nelineární pružina – $\boxed{\text{elastická}}$



$$W = \int_0^{\Delta} F(u)\,du$$

2





$$W = \frac{1}{2} F \cdot \delta = \frac{1}{2} F \cdot \frac{Fl}{ES} =$$

$$= \frac{1}{2} \frac{F^2 l}{ES}$$ → v deformované tyči je práce vnější síly uložena jako def. en. Uvolní se přiodlehčení.

$U = W$

Hustota deformační energie (na jednotku objemu)

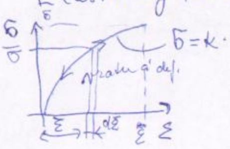
$$\lambda = \frac{U}{V} = \frac{1}{Sl} \cdot \frac{1}{2} \frac{F^2 l}{ES} = \frac{1}{2E}\left(\frac{F}{S}\right)^2 = \frac{1}{2} \frac{\sigma^2}{E} =$$

$= \frac{1}{2} \sigma \cdot \varepsilon$ → plocha v zatěž. diagr.

$$\lambda = \int_0^{\varepsilon} \bar{\sigma}\,d\bar{\varepsilon} = \int_0^{\varepsilon} E \cdot \bar{\varepsilon}\,d\bar{\varepsilon} = \frac{1}{2} E \cdot \varepsilon^2$$

$$U = \int \lambda\,dV$$

Elastický nelineární (materiál) tyče

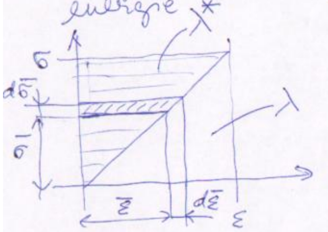


$$\lambda = \int_0^{\varepsilon} \bar{\sigma} \cdot d\bar{\varepsilon} = \int_0^{\varepsilon} k \cdot \bar{\varepsilon}^{\frac{1}{2}}\,d\bar{\varepsilon} =$$

$$= \frac{2}{3} k \cdot \varepsilon^{\frac{3}{2}} = \frac{2}{3} \sigma \cdot \varepsilon$$

3

Komplementární deformační energie λ^*

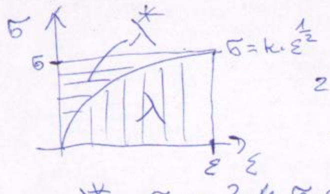


$$\int_0^{\sigma} \bar{\varepsilon}(\bar{\sigma})\,d\bar{\sigma} =$$

$$= \int_0^{\sigma} \frac{1}{K} \cdot \bar{\sigma}\,d\bar{\sigma} = \frac{1}{K} \cdot \frac{\sigma^2}{2} =$$

$$= \frac{1}{2} \sigma \cdot \varepsilon = \frac{1}{2} K \varepsilon^2$$

při lin. záv. $\lambda = \lambda^*$



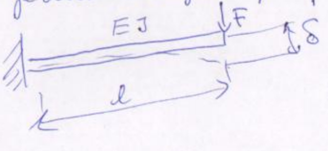
z předch.

$$\lambda = \frac{2}{3} \sigma \varepsilon$$

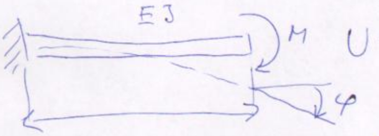
$$\lambda^* = \sigma \cdot \varepsilon - \frac{2}{3} \sigma \cdot \varepsilon = \frac{1}{3} \sigma \cdot \varepsilon$$

4

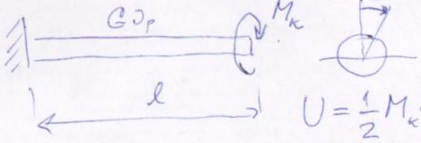
Podobně bychom mohli stanovit deformační energii v některých jednodušších případech



$$U = \frac{1}{2} F \cdot \delta = \frac{1}{2} F \cdot \frac{Fl^3}{3EJ}$$

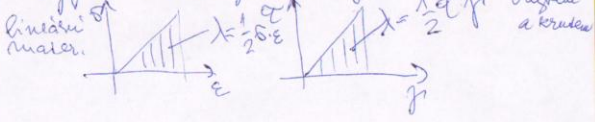


$$U = \frac{1}{2} M \cdot \varphi = \frac{1}{2} M \cdot \frac{Ml}{EJ}$$



$$U = \frac{1}{2} M_k \cdot \vartheta = \frac{1}{2} M_k \frac{M_k l}{G J_p}$$

Deformační energie u součástí typu dlouhých štíhlých tyčí zatížených tahem, ohybem a krutem



Tah O.K. (5)

$$U=\int_{(l)}\left(\int_{(S)}\lambda(x)\,dS\right)dx=\int_{(l)}\left(\int_{(S)}\frac{\sigma^2(x)}{2E}\,dS\right)dx=$$

$$\sigma(x)=\frac{N(x)}{S}$$

$$=\frac{1}{2E}\int_{(l)}\left\{\frac{N(x)}{2SE}\right\}^2\cdot S\,dx$$

$$\int_{(S)}dS=S$$

Ohyb

$$\sigma(x,z)=\frac{M(x)}{J_y}\cdot z$$

$$U=\frac{1}{2E}\int_{(l)}\left(\int_{(S)}\left(\frac{M_o(x)}{J_y}z\right)^2dS\right)dx=\quad\int_{(S)}z^2\,dS=J_y$$

$$=\int_{(l)}\frac{M_o^2(x)\,dx}{2EJ_y}$$

$$\int_{(S)}r^2\,dS=J_p$$

Krut

$$U=\frac{1}{2G}\int_{(l)}\left(\int_{(S)}\left(\frac{M_k(x)}{J_p}\cdot r\right)^2dS\right)dx$$

$$=\int_{(l)}\frac{M_k^2(x)\,dx}{2GJ_p}$$

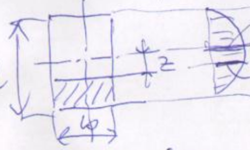
(6)

Energie def. od příčné síly u ohýbaných tyčí

napětí $\tau_{xz}=\dfrac{T(x)\cdot\bar{S}_y}{t(x)\cdot J_y}$

$\bar{S}_y$ = stat. mom. oddělené plochy
$t(x)$ šířka v daném místě

Pro obdél. průřez



$$\tau_{xz}(z)=\frac{T(x)\frac{1}{2}b\cdot\left[\left(\frac{h}{2}\right)^2-z^2\right]}{b\cdot J_y}$$

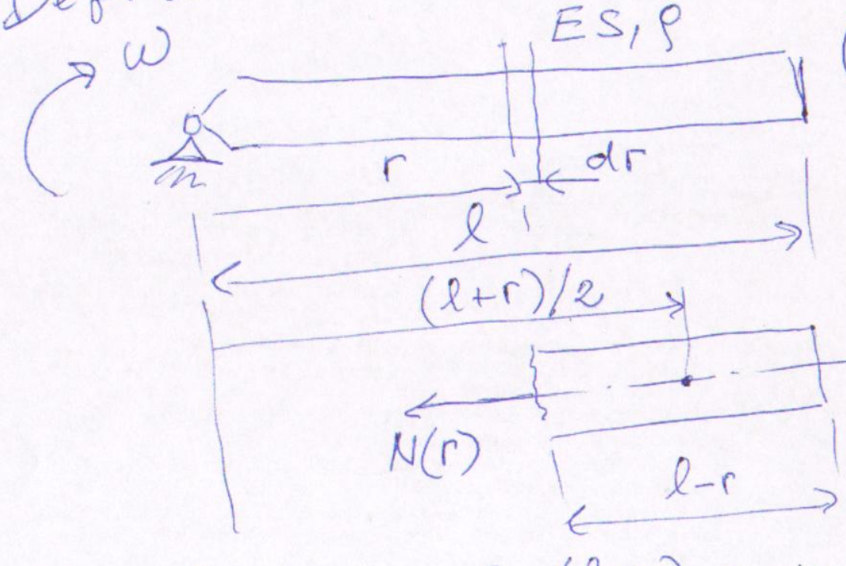
$$U_T=\int_{(l)}\left(\int_{(S)}\left(\frac{T(x)}{J_y}\right)^2\cdot\frac{1}{2G}\frac{1}{4}\left[\left(\frac{h}{2}\right)^2-z^2\right]^2dS\right)dx$$

$$=\int_{(l)}\frac{24}{45}\frac{T^2(x)\,dx}{2GS}$$

$\beta=\frac{6}{5}$!!

Koef. rozdílné průř.

Deformační energie v rot. rameni



7

$$N(r) = (l-r)\cdot S\cdot \rho\cdot \omega^2 \cdot \frac{(l+r)}{2} \quad [N]$$

$$\sigma(r) = \frac{l^2-r^2}{2}\rho\,\omega^2 \quad [Pa]$$

$$\lambda(r) = \frac{\sigma^2(r)}{2E} \quad [J\,m^{-3}]$$

$$\frac{8l^5}{15}$$

$$U = \int_{(V)} \lambda(r)\,dV = \frac{S}{2E}\,\frac{\rho^2\omega^4}{4}\int_0^l (l^2-r^2)^2\,dr = \frac{S\rho^2\omega^4 l^5}{15E}\ [J] \quad Nm$$

Kin. en.

$$dK = \frac{1}{2}\rho S\,dr\,(r\omega)^2$$

$$K = \int_0^l dK = \frac{1}{2}\rho S\omega^2\frac{l^3}{3}$$

$$kg\,m^{-3}\,m^2\,s^{-2}\,m^3 = N\cdot m \quad \text{o.k.}$$

$$\frac{K}{U} = \frac{\frac{1}{2}\cdot 15E}{\rho\omega^2 l^2}$$

$$\frac{K}{U} = \frac{\frac{5}{2}E}{\rho\omega^2 l^2} = \frac{5}{4}\frac{E}{\sigma_{max}}$$

$$\sigma_{max} = \frac{1}{2}\rho\omega^2 l^2$$