Přehled předchozích závislostí:

Deformační energie v tyčích zatížených:

1) tahem $U = \int_{(l)} \frac{N^2(x)\,dx}{2ES}$

2) ohybem $U = \int_{(l)} \frac{M_o^2(x)\,dx}{2EJ_y}$

3) krutem $U = \int_{(l)} \frac{M_k^2(x)\,dx}{2GJ_p}$

Castiglianovy věty:

1.C.V. $q_i = \frac{\partial U^*}{\partial Q_i}$

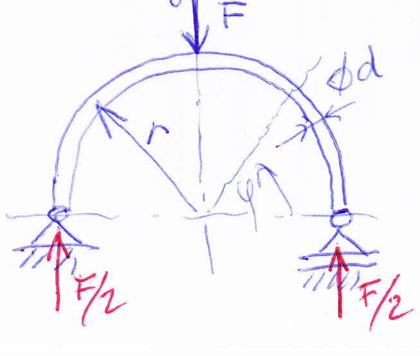
q_i = zobec. posuv
Q_i = zobec. síla
U^* = doplň. def. energ.

2.C.V. $Q_i = \frac{\partial U}{\partial q_i}$

3.C.V. (věta o minimu def. en.) $\frac{\partial U}{\partial X_i} = 0$ v lineárních případech

X = staticky neurčitá zobec. síla

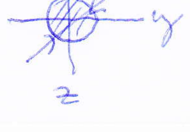
Příklad – tenký křivý prut – uvažujeme pouze def. energii od ohybu nebo krutu



úkolem je vypočítat průhyb pod silou F

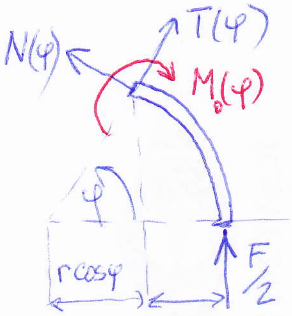
$w = \frac{\partial U}{\partial F}$ (materiál je lineární a nevznikají velké deformace)

$U = 2\int_0^{\pi/2} \frac{M_o^2(\varphi)\, r\, d\varphi}{2\,EJ_y}$



$d \ll r$
tenký prut

Vedeme myšlený řez pod úhlem φ

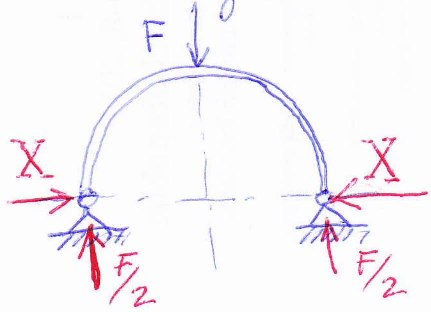


$$M_o(\varphi) = \frac{F}{2} r(1-\cos\varphi) \qquad \frac{\partial M_o(\varphi)}{\partial F} = \frac{1}{2} r(1-\cos\varphi)$$

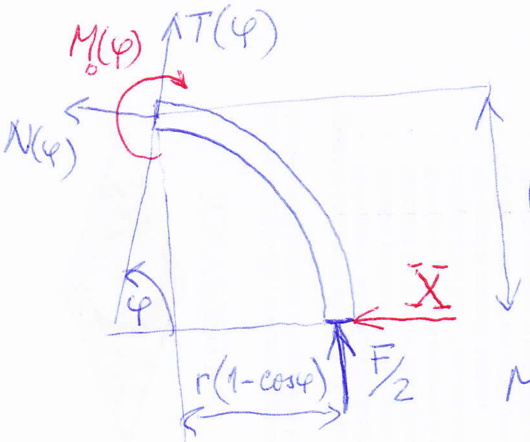
$$\frac{\partial U}{\partial F} = 2\frac{\partial}{\partial F}\int_0^{\pi/2} \frac{M_o^2(\varphi)\, r\, d\varphi}{2EJ} = 2\int_0^{\pi/2}\frac{\frac{\partial}{\partial F}\left(M_o^2(\varphi)\right) r\, d\varphi}{2EJ} =$$

$$= 2\int_0^{\pi/2} \frac{2M_o(\varphi)\cdot\frac{\partial M_o(\varphi)}{\partial F}}{2EJ}\, r\, d\varphi = \frac{2}{EJ}\frac{F\cdot r^3}{4}\int_0^{\pi/2}(1-\cos\varphi)^2\, d\varphi$$

Staticky neurčitý případ – podpory se nemohou posunout, vzniká stat. neurč. síla X



věta o minimu: $\frac{\partial U}{\partial X} = 0 \Rightarrow X$



$$M_o(\varphi) = \frac{F}{2} r(1-\cos\varphi) - X\, r\sin\varphi$$

$$\frac{\partial M_o(\varphi)}{\partial X} = -r\sin\varphi$$

$$0 = \frac{\partial U}{\partial X} = \frac{2}{EJ}\int_0^{\pi/2} M_o(\varphi)\cdot\frac{\partial M_o(\varphi)}{\partial X}\cdot r\, d\varphi = \frac{2}{EJ}\int_0^{\pi/2}\left[\frac{F}{2}r(1-\cos\varphi) - X\cdot r\sin\varphi\right]\cdot$$

$$\cdot(-r\sin\varphi)\cdot r\, d\varphi = 0 \Rightarrow X$$

Deformační energie v hustě vinuté pružině

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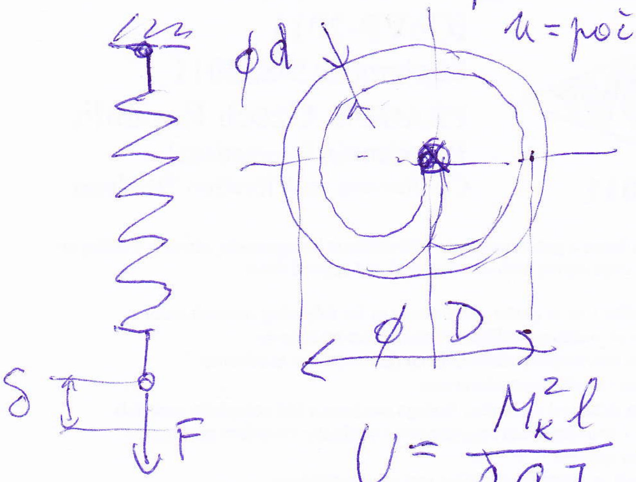


n = počz záv. G modul vesmu

$$M_K = F \cdot \frac{D}{2}$$

Délka pružiny

$$l = \pi D \cdot n$$

$$J_p = \frac{\pi d^4}{32}$$

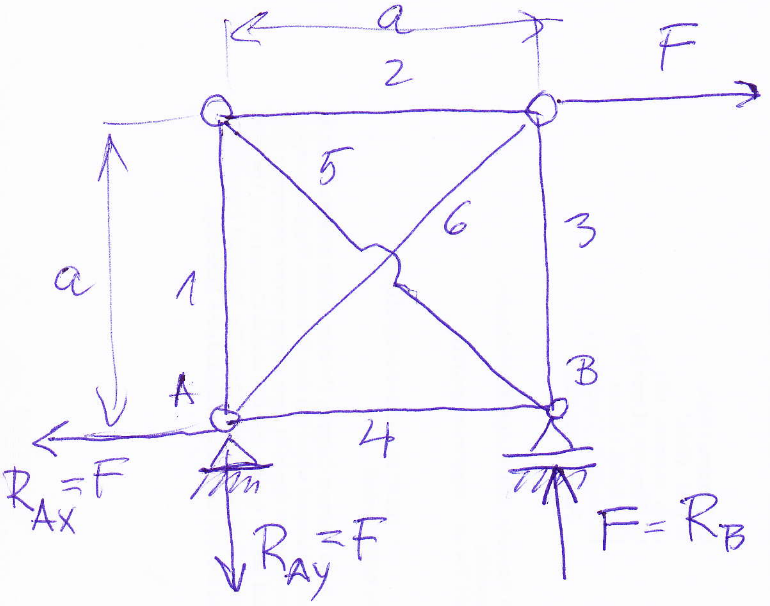
$$U = \frac{M_K^2 l}{2 G J_p} = \frac{(F \frac{D}{2})^2 l}{2 G J_p}$$

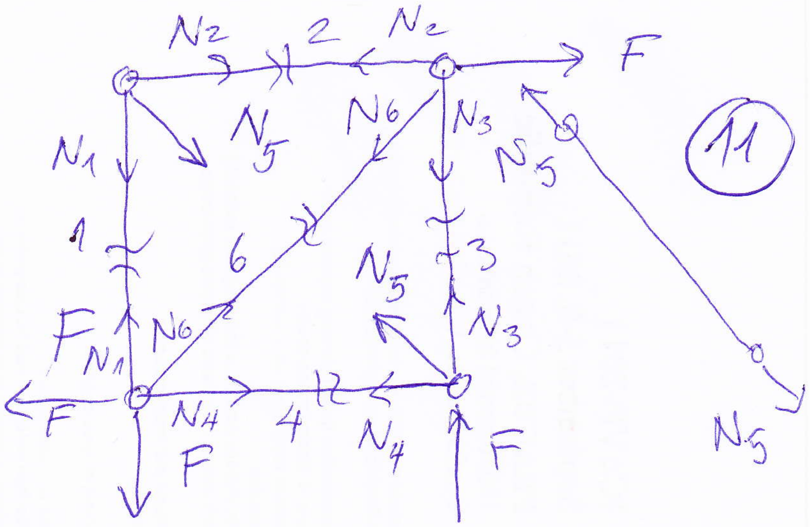
zároveň však: $U = F \cdot \frac{1}{2} \delta$

Prosazení pružiny

$$\frac{F^2 \frac{D^2}{4} \cdot l}{\cancel{2} G \frac{\pi d^4}{32}} = \frac{1}{\cancel{2}} F \cdot \delta$$

$$\delta = \frac{8 F D^2 \cdot \pi D n}{G d^4}$$





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$$N_1 = N_2$$

$$\cancel{2} N_1 \cdot \frac{\sqrt{2}}{\cancel{2}} + N_5 = 0$$

$$N_1 = -\frac{1}{\sqrt{2}} N_5 = -\frac{\sqrt{2}}{2} N_5$$

$$-\frac{\sqrt{2}}{2} N_5 + \frac{\sqrt{2}}{2} N_6 - F = 0$$

$$N_6 = \frac{2}{\sqrt{2}}\left(F + \frac{\sqrt{2}}{2} N_5\right)$$

i	N_i	l_i	$\frac{\partial N_i}{\partial N_5}$
1	$-\frac{\sqrt{2}}{2} N_5$	a	$-\frac{\sqrt{2}}{2}$
2	$-\frac{\sqrt{2}}{2} N_5$	a	$-\frac{\sqrt{2}}{2}$
3	$-F-\frac{\sqrt{2}}{2} N_5$	a	$-\frac{\sqrt{2}}{2}$
4	$-\frac{\sqrt{2}}{2} N_5$	a	$-\frac{\sqrt{2}}{2}$
5	N_5	$\sqrt{2} a$	1
6	$\sqrt{2} F + N_5$	$\sqrt{2} a$	1

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$$N_3 + \frac{\sqrt{2}}{2} N_6 = 0$$

$$N_3 = -\frac{\sqrt{2}}{2}\left(\sqrt{2} F + N_5\right) = -F - \frac{\sqrt{2}}{2} N_5$$

$$N_4 = -\frac{\sqrt{2}}{2} N_5$$

$$\frac{\partial U}{\partial N_5} = 0 \qquad \frac{\partial U}{\partial N_5} = \sum \frac{N_i l_i}{E_i S_i} \cdot \frac{\partial N_i}{\partial N_5} = 0$$

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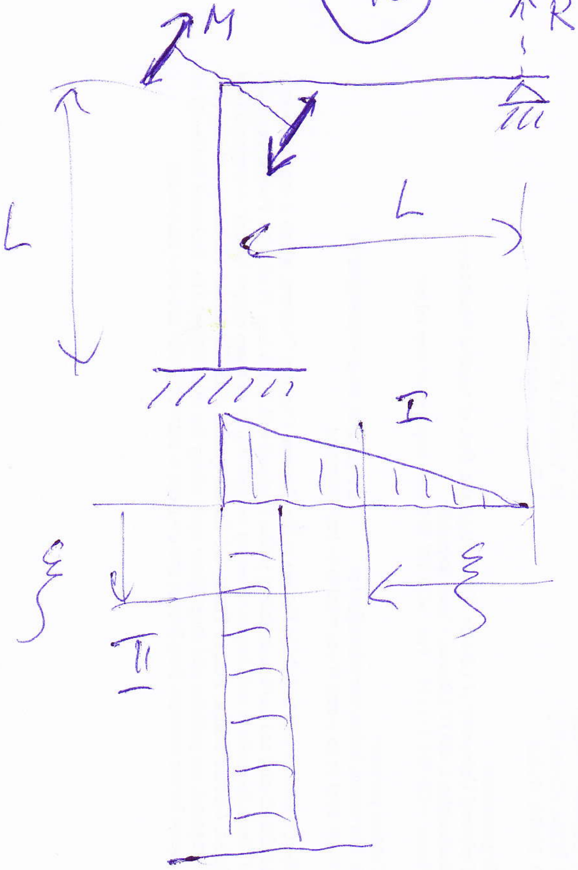
? maximální vnitřní moment



		int	$\frac{\partial M}{\partial R}$
I	$M_I(\xi) = R\cdot\xi$	0,L	ξ
II	$M_{II}(\xi) = R\cdot L - M$	0,L	L

$$\frac{\partial U}{\partial R} = 0 \qquad U = U_1 + U_2 = \int_{(\ell_1)} \frac{M_I^2(\xi)\,d\xi}{2EJ} + \int_{(\ell_2)} \frac{M_{II}^2(\xi)\,d\xi}{2EJ}$$

$$\frac{\partial U}{\partial R} = \int_{(\ell_1)} \frac{M_I(\xi)\cdot\frac{\partial M_I}{\partial R}}{EJ}\,d\xi + \int_{(\ell_2)} \frac{M_{II}(\xi)\frac{\partial M_{II}}{\partial R}}{EJ}\,d\xi$$

$$\frac{\partial U}{\partial R} = \int_0^L \frac{R\cdot\xi\cdot\xi\,d\xi}{EJ} + \int_0^L \frac{(RL-M)\cdot L\,d\xi}{EJ} \qquad 0 = \frac{RL^3}{3EJ} + \frac{RL^3 - ML^2}{EJ}$$