CHAPTER 5

TORSION OF NON-CIRCULAR AND THIN-WALLED SECTIONS

Summary

For torsion of *rectangular sections* the maximum shear stress τ_{max} and angle of twist θ are given by

$$\tau_{max} = \frac{T}{k_1 d b^2}$$

$$\frac{\theta}{L} = \frac{T}{k_2 d b^3 G}$$

k_1 and k_2 being two constants, their values depending on the ratio d/b and being given in Table 5.1.

For *narrow rectangular sections*, $k_1 = k_2 = \frac{1}{3}$.

Thin-walled open sections may be considered as combinations of narrow rectangular sections so that

$$\tau_{max} = \frac{T}{\Sigma k_1 d b^2} = \frac{3T}{\Sigma d b^2}$$

$$\frac{\theta}{L} = \frac{T}{\Sigma k_2 d b^3 G} = \frac{3T}{G \Sigma d b^3}$$

The relevant formulae for other non-rectangular, non-tubular solid shafts are given in Table 5.2.

For *thin-walled closed sections* the stress at any point is given by

$$\tau = \frac{T}{2At}$$

where A is the area enclosed by the median line or mean perimeter and t is the thickness. The maximum stress occurs at the point where t is a minimum.

The angle of twist is then given by

$$\theta = \frac{TL}{4A^2 G} \int \frac{ds}{t}$$

which, for *tubes of constant thickness*, reduces to

$$\frac{\theta}{L} = \frac{Ts}{4A^2 G t} = \frac{\tau s}{2AG}$$

where s is the length or perimeter of the median line.

Thin-walled cellular sections may be solved using the concept of constant shear flow $q(=\tau t)$, bearing in mind that the angles of twist of all cells or constituent parts are assumed equal.

5.1. Rectangular sections

Detailed analysis of the torsion of non-circular sections which includes the warping of cross-sections is beyond the scope of this text. For *rectangular shafts*, however, with longer side d and shorter side b, it can be shown by experiment that the maximum shearing stress occurs at the centre of the longer side and is given by

$$\tau_{max} = \frac{T}{k_1 d b^2} \tag{5.1}$$

where k_1 is a constant depending on the ratio d/b and given in Table 5.1 below.

Table 5.1. Table of k_1 and k_2 values for rectangular sections in torsion[a].

d/b	1.0	1.5	1.75	2.0	2.5	3.0	4.0	6.0	8.0	10.0	∞
k_1	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
k_2	0.141	0.196	0.214	0.229	0.249	0.263	0.281	0.299	0.307	0.313	0.333

[a] S. Timoshenko, *Strength of Materials*, Part I, *Elementary Theory and Problems*, Van Nostrand, New York.

The essential difference between the shear stress distributions in circular and rectangular members is illustrated in Fig. 5.1, where the shear stress distribution along the major and minor axes of a rectangular section together with that along a "radial" line to the corner of the section are indicated. The maximum shear stress is shown at the centre of the longer side, as noted above, and the stress at the corner is zero.

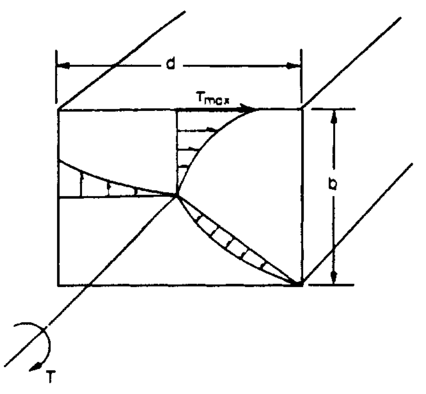


Fig. 5.1. Shear stress distribution in a solid rectangular shaft.

The angle of twist per unit length is given by

$$\frac{\theta}{L} = \frac{T}{k_2 d b^3 G} \tag{5.2}$$

k_2 being another constant depending on the ratio d/b and also given in Table 5.1.

In the absence of Table 5.1, however, it is possible to reduce the above equations to the following *approximate* forms:

$$\tau_{\max} = \frac{T}{db^2}\left[3 + 1.8\frac{b}{d}\right] = \frac{T}{db^3}[3d + 1.8b] \tag{5.3}$$

and

$$\theta = \frac{42TLJ}{GA^4} = \frac{42TLJ}{Gd^4b^4} \tag{5.4}$$

where A is the cross-sectional area of the section ($= bd$) and $J = (bd/12)(b^2 + d^2)$.

5.2. Narrow rectangular sections

From Table 5.1 it is evident that as the ratio d/b increases, i.e. the rectangular section becomes longer and thinner, the values of constants k_1 and k_2 approach 0.333. Thus, for narrow rectangular sections in which $d/b > 10$ both k_1 and k_2 are assumed to be 1/3 and eqns. (5.1) and (5.2) reduce to

$$\tau_{\max} = \frac{3T}{db^2} \tag{5.5}$$

$$\frac{\theta}{L} = \frac{3T}{db^3G} \tag{5.6}$$

5.3. Thin-walled open sections

There are many cases, particularly in civil engineering applications, where rolled steel or extruded alloy sections are used where some element of torsion is involved. In most cases the sections consist of a combination of rectangles, and the relationships given in eqns. (5.1) and (5.2) can be adapted with reasonable accuracy provided that:

(a) the sections are "open", i.e. angles, channels. T-sections, etc., as shown in Fig. 5.2;
(b) the sections are thin compared with the other dimensions.

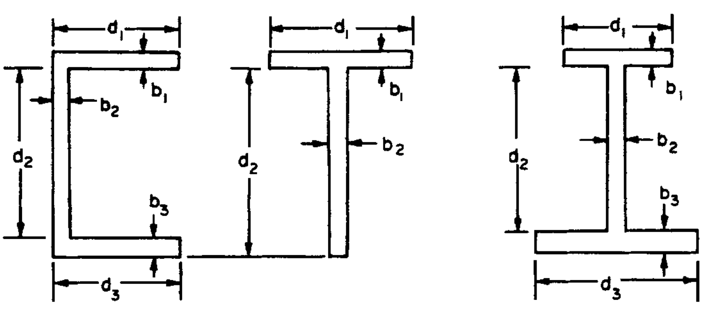


Fig. 5.2. Typical thin-walled open sections.

For such sections eqns. (5.1) and (5.2) may be re-written in the form

$$\tau_{\max} = \frac{T}{k_1 d b^2} = \frac{T}{Z'} \tag{5.7}$$

and

$$\frac{\theta}{L} = \frac{T}{k_2 d b^3 G} = \frac{T}{J_{\text{eq}} G} \tag{5.8}$$

where Z' is the torsion section modulus

$$= Z' \text{ web} + Z' \text{ flanges} = k_1 d_1 b_1^2 + k_1 d_2 b_2^2 + \ldots \text{etc.}$$
$$= \Sigma k_1 d b^2$$

and J_{eq} is the "effective" polar moment of area or "equivalent J" (see §5.7)

$$= J_{\text{eq}} \text{ web} + J_{\text{eq}} \text{ flanges} = k_2 d_1 b_1^3 + k_2 d_2 b_2^3 + \cdots \text{etc.}$$
$$= \Sigma k_2 d b^3$$

i.e.

$$\boldsymbol{\tau_{\max} = \frac{T}{\sum k_1 d b^2}} \tag{5.9}$$

and

$$\boldsymbol{\frac{\theta}{L} = \frac{T}{G \sum k_2 d b^3}} \tag{5.10}$$

and for d/b ratios in excess of 10, $k_1 = k_2 = \frac{1}{3}$, so that

$$\boldsymbol{\tau_{\max} = \frac{3T}{\sum d b^2}} \tag{5.11}$$

$$\boldsymbol{\frac{\theta}{L} = \frac{3T}{G \sum d b^3}} \tag{5.12}$$

To take account of the stress concentrations at the fillets of such sections, however, Timoshenko and Young† suggest that the maximum shear stress as calculated above is multiplied by the factor

$$\left[1 + \frac{b}{4a}\right]$$

(Figure 5.3). This has been shown to be fairly reliable over the range $0 < a/b < 0.5$. In the event of sections containing limbs of different thicknesses the largest value of b should be used.

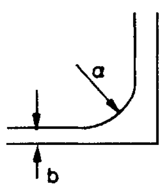


Fig. 5.3.

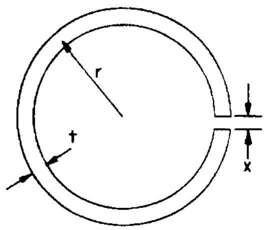
† S. Timoshenko and A.D. Young, *Strength of Materials*, Van Nostrand, New York, 1968 edition.

5.4. Thin-walled split tube

The thin-walled split tube shown in Fig. 5.4 is considered to be a special case of the thin-walled open type of section considered in §5.3. It is therefore treated as an equivalent rectangle with a longer side d equal to the circumference (less the gap), and a width b equal to the thickness.

Then
$$\tau_{max} = \frac{T}{k_1 db^2}$$

and
$$\frac{\theta}{L} = \frac{T}{k_2 db^3 G}$$



d = mean circumference = $2\pi r$

Fig. 5.4. Thin tube with longitudinal split.

where k_1 and k_2 for thin-walled tubes are usually equal to $\frac{1}{3}$.

It should be noted here that the presence of even a very small cut or gap in a thin-walled tube produces a torsional stiffness (torque per unit angle of twist) very much smaller than that for a complete tube of the same dimensions.

5.5. Other solid (non-tubular) shafts

Table 5.2 (see p. 146) indicates the relevant formulae for maximum shear stress and angle of twist of other standard non-circular sections which may be encountered in practice.

Approximate angles of twist for other solid cross-sections may be obtained by the substitution of an elliptical cross-section of the same area A and the same polar second moment of area J. The relevant equation for the elliptical section in Table 5.2 may then be applied.

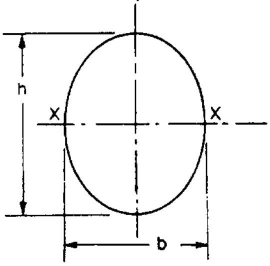
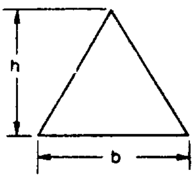
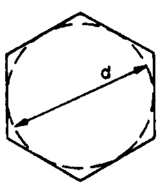
Alternatively, a very powerful procedure which applies for all solid sections, however irregular in shape, utilises a so-called "inscribed circle" procedure described in detail by Roark†. The procedure is equally applicable to thick-walled standard T, I and channel sections and is outlined briefly below:

Inscribed circle procedure

Roark shows that the maximum shear stress which is set up when any solid section is subjected to torque occurs at, or very near to, one of the points where the largest circle which

† R.J. Roark and W.C. Young, *Formulas for Stress & Strain*, 5th edn. McGraw-Hill, Kogakusha.

Table 5.2[a].

Cross-section	Maximum shear stress	Angle of twist per unit length
Elliptic 	$\dfrac{16T}{\pi b^2 h}$ at end of minor axis XX	$\dfrac{4\pi^2 TJ}{A^4 G}$
	where $J = \dfrac{\pi}{64}[bh^3 + hb^3]$ and A is the area of cross-section $= \pi bh/4$	
Equilateral triangle 	$\dfrac{20T}{b^3}$ at the middle of each side	$\dfrac{46.2T}{b^4 G}$
Regular hexagon 	$\dfrac{T}{0.217\ Ad}$	$\dfrac{T}{0.133\ Ad^2 G}$
	where d is the diameter of inscribed circle and A is the cross-sectional area	

[a] From S. Timoshenko, *Strength of Materials*, Part II, *Advanced Theory and Problems*, Van Nostrand, New York, p. 235. Approximate angles of twist for other solid cross-sections may be obtained by the substitution of an equivalent elliptical cross-section of the same area A and the same polar second moment of area J. The relevant equation for the elliptical section in Table 5.2 may then be applied.

can be constructed within the cross-section touches the section boundary – see Fig. 5.5. Normally it occurs at the point where the curvature of the boundary is algebraically the least, convex curvatures being taken as positive and concave or re-entrant curvatures negative.

The maximum shear stress is then obtained from either:

$$\tau_{\max} = \left(\frac{G\theta}{L}\right) C \quad \text{or} \quad \tau_{\max} = \left(\frac{\tau}{K}\right) C$$

where, *for positive curvatures* (i.e. straight or convex boundaries),

$$C = \frac{D}{1 + \dfrac{\pi^2 D^4}{16A^2}}\left[1 + 0.15\left(\frac{\pi^2 D^4}{16A^2} - \frac{D}{2r}\right)\right]$$

with D = diameter of the largest inscribed circle,

r = radius of curvature of boundary at selected position (positive),

A = cross-sectional area of section,

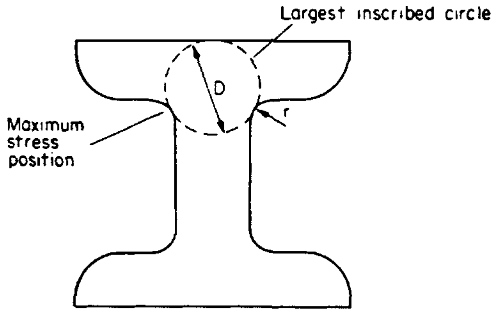


Fig. 5.5. Inscribed circle stress evaluation procedure.

or, *for negative curvatures* (concave or re-entrant boundaries):

$$C = \frac{D}{1 + \dfrac{\pi^2 D^4}{16A^2}}\left[1 + \left\{0.118\log_e\left(1 - \frac{D}{2r}\right) - 0.238\frac{D}{2r}\right\}\tanh\frac{2\phi}{\pi}\right]$$

with ϕ = angle through which a tangent to the boundary rotates in travelling around the re-entrant position (radians) and r being taken as negative.

For standard thick-walled open sections such as T, I, Z, angle and channel sections Roark also introduces formulae for angles of twist based upon the same inscribed circle procedure parameters.

5.6. Thin-walled closed tubes of non-circular section (Bredt–Batho theory)

Consider the thin-walled closed tube shown in Fig. 5.6 subjected to a torque T about the Z axis, i.e. in a transverse plane. Both the cross-section and the wall thickness around the periphery may be irregular as shown, but for the purposes of this simplified treatment it must be assumed that the thickness does not vary along the length of the tube. Then, if τ is the shear stress at B and τ' is the shear stress at C (where the thickness has increased to t') then, from the equilibrium of the complementary shears on the sides AB and CD of the element shown, it follows that

$$\tau t\, dz = \tau' t'\, dz$$

$$\tau t = \tau' t'$$

i.e. *the product of the shear stress and the thickness is constant* at all points on the periphery of the tube. This constant is termed the *shear flow* and denoted by the symbol q (shear force per unit length).

Thus $$q = \tau t = \text{constant} \qquad (5.13)$$

The quantity q is termed the shear flow because if one imagines the inner and outer boundaries of the tube section to be those of a channel carrying a flow of water, then, provided that the total quantity of water in the system remains constant, the quantity flowing past any given point is also constant.

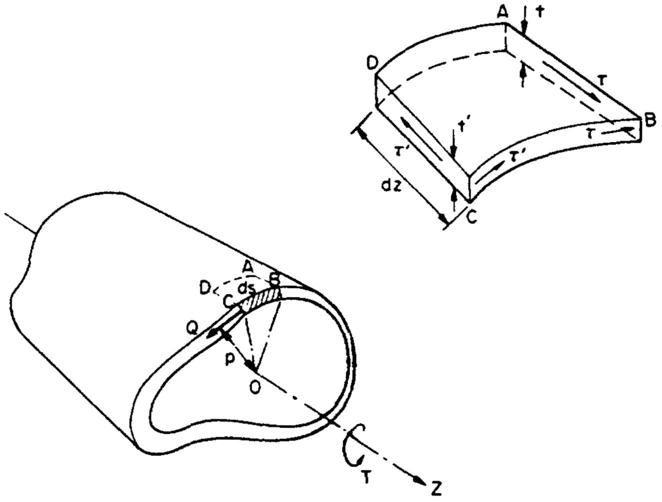


Fig. 5.6. Thin-walled closed section subjected to axial torque.

At any point, then, the shear force Q on an element of length ds is $Q = \tau t\,ds = q\,ds$ and the shear stress is q/t.

Consider now, therefore, the element BC subjected to the shear force $Q = q\,ds = \tau t\,ds$.

The moment of this force about O

$$= dT = Qp$$

where p is the perpendicular distance from O to the force Q.

$$\therefore \qquad dT = q\,ds\,p$$

Therefore the moment, or torque, for the whole section

$$= \int qp\,ds = q\int p\,ds$$

But the area $COB = \frac{1}{2}$ base $\times$ height $= \frac{1}{2}p\,ds$

i.e. $$dA = \tfrac{1}{2}p\,ds \quad \text{or} \quad 2dA = p\,ds$$

$$\therefore \qquad \text{torque } T = 2q\int dA$$

$$T = 2qA \tag{5.14}$$

where A is the area enclosed within the median line of the wall thickness.

Now, since

$$q = \tau t$$

it follows that $$T = 2\tau t A$$

or $$\tau = \frac{T}{2At} \tag{5.15}$$

where t is the thickness at the point in question.

It is evident, therefore, that *the maximum shear stress* in such cases *occurs at the point of minimum thickness*.

Consider now an axial strip of the tube, of length L, along which the thickness and hence the shear stress is constant. The shear strain energy *per unit volume* is given by

$$U = \int \frac{\tau^2}{2G}$$

Thus, with thickness t, width ds and hence $V = tL\,ds$

$$\begin{aligned} U &= \int \frac{\tau^2}{2G} tL\,ds \\ &= \int \left(\frac{T}{2At}\right)^2 \frac{tL}{2G}\,ds \\ &= \frac{T^2 L}{8A^2 G} \int \frac{ds}{t} \end{aligned}$$

But the energy stored equals the work done $= \frac{1}{2}T\theta$.

$$\therefore \qquad \frac{1}{2}T\theta = \frac{T^2 L}{8A^2 G} \int \frac{ds}{t}$$

The angle of twist of the tube is therefore given by

$$\boldsymbol{\theta = \frac{TL}{4A^2G} \int \frac{ds}{t}}$$

For *tubes of constant thickness* this reduces to

$$\boldsymbol{\theta = \frac{TLs}{4A^2Gt} = \frac{\tau Ls}{2AG}} \tag{5.16}$$

where s is the perimeter of the median line.

The above equations must be used with care and do not apply to cases where there are abrupt changes in thickness or re-entrant corners.

For closed sections which have constant thickness over specified lengths but varying from one part of the perimeter to another:

$$\frac{\theta}{L} = \frac{T}{4A^2G}\left[\frac{s_1}{t_1} + \frac{s_2}{t_2} + \frac{s_3}{t_3} + \cdots \text{etc.}\right]$$

5.7. Use of "equivalent J" for torsion of non-circular sections

The simple torsion theory for circular sections can be written in the form:

$$\frac{\theta}{L} = \frac{T}{GJ}$$

and, as stated on page 143, it is often convenient to express the twist of non-circular sections in similar form:

i.e.
$$\frac{\theta}{L} = \frac{T}{GJ_{eq}}$$

where J_{eq} is the "*equivalent J*" or "*effective polar moment of area*" for the section in question.

Thus, *for open sections:*
$$\frac{\theta}{L} = \frac{T}{\Sigma k_2 db^3 G} = \frac{T}{GJ_{eq}}$$

with $J_{eq} = \Sigma k_2 db^3 \quad (= \frac{1}{3}\Sigma db^3 \text{ for } d/b > 10)$.

Similarly, *for square tubes of closed section:*
$$\frac{\theta}{L} = \frac{TLs}{4A^2Gt} = \frac{T}{G[4A^2t/s]} = \frac{T}{GJ_{eq}}$$

and $J_{eq} = 4A^2t/s$.

The *torsional stiffness* of any section, i.e. the ratio of torque divided by angle of twist per unit length, is then directly given by the value of GJ or GJ_{eq} i.e.

$$\textbf{Stiffness} = \frac{T}{\theta/L} = GJ \text{ (or } GJ_{eq}).$$

5.8. Thin-walled cellular sections

The Bredt–Batho theory developed in the previous section may be applied to the solution of problems involving cellular sections of the type shown in Fig. 5.7.

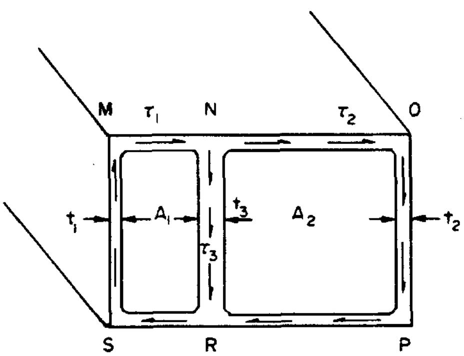


Fig. 5.7. Thin-walled cellular section.

Assume the length *RSMN* is of constant thickness t_1 and subjected therefore to a constant shear stress τ_1. Similarly, *NOPR* is of thickness t_2 and stress τ_2 with *NR* of thickness t_3 and stress τ_3.

Considering the equilibrium of complementary shear stresses on a longitudinal section at N, it follows that
$$\tau_1 t_1 = \tau_2 t_2 + \tau_3 t_3 \tag{5.17}$$

Alternatively, this equation may be obtained considering the arrows shown to be directions of shear flow $q(=\tau t)$. At N the flow q_1 along MN divides into q_2 along NO and q_3 along NR,

i.e. $$q_1 = q_2 + q_3$$

or $$\tau_1 t_1 = \tau_2 t_2 + \tau_3 t_3 \quad \text{(as before)}$$

The total torque for the section is then found as the sum of the torques on the two cells by application of eqn. (5.14) to the two cells and adding the result,

i.e. $$T = 2q_1A_1 + 2q_2A_2$$

$$T = 2(\tau_1 t_1 A_1 + \tau_2 t_2 A_2) \tag{5.18}$$

Also, since the angle of twist will be common to both cells, applying eqn. (5.16) to each cell gives

$$\theta = \frac{L}{2G}\left(\frac{\tau_1 s_1 + \tau_3 s_3}{A_1}\right) = \frac{L}{2G}\left(\frac{\tau_2 s_2 - \tau_3 s_3}{A_2}\right)$$

where s_1, s_2 and s_3 are the median line perimeters $RSMN$, $NOPR$ and NR respectively.

The negative sign appears in the final term because the shear flow along NR for this cell opposes that in the remainder of the perimeter.

$$\therefore \qquad \frac{2G\theta}{L} = \frac{1}{A_1}(\tau_1 s_1 + \tau_3 s_3) = \frac{1}{A_2}(\tau_2 s_2 - \tau_3 s_3) \tag{5.19}$$

5.9. Torsion of thin-walled stiffened sections

The stiffness of any section has been shown above to be given by its value of GJ or GJ_{eq}.

Consider, therefore, the rectangular polymer extrusion of simple symmetrical cellular constructions shown in Fig. 5.8(a). The shear flow in each cell is indicated.

At A $$q_1 = q_2 + q_3.$$

But because of symmetry q_1 must equal q_3 $\therefore\ q_2 = 0$;
i.e., for a symmetrical cellular thin-walled member there is no shear carried by the central web and therefore as far as stiffness of the section is concerned the web can be ignored.

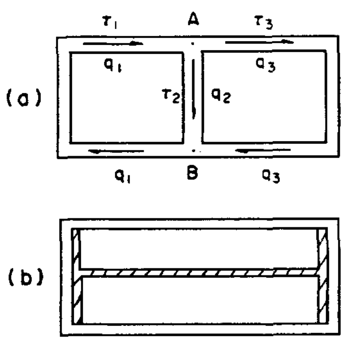


Fig. 5.8(a). Polymer cellular section with symmetrical cells. (b) Polymer cell with central web removed but reinforced by steel I section.

$\therefore$ Stiffness of complete section, from eqn. (5.16)

$$= GJ_E = \frac{4A^2t}{s}G$$

where A and s are the area and perimeter of the complete section.

Now since G of the polymer is likely to be small, the stiffness of the section, and its resistance to applied torque, will be low. It can be reinforced by metallic insertions such as that of the I section shown in Fig. 5.8(b).

For the I section, from eqn. (5.8)

$$GJ_E = G\Sigma k_2 db^3$$

and the value represents the increase in stiffness presented by the compound section.

Stress conditions for limiting twist per unit lengths are then given by:
For the tube

$$T = GJ_E(\theta/L) = 2At\tau$$

$$\therefore (\theta/L)_{\max} = \frac{2At}{GJ_E} \cdot \tau_{\max}$$

and for the I section

$$T = GJ_E(\theta/L) = (\Sigma k_2 db^3 G)\theta/L$$

or

$$T = (\Sigma k_1 db^2)\tau$$

$$\therefore \quad (\theta/L)_{\max} = \frac{\Sigma k_1 d}{Gb\Sigma k_2 d} \cdot \tau_{\max}$$

Usually (but not always) this would be considerably greater than that for the polymer tube, making the tube the controlling design factor.

5.10. Membrane analogy

It has been stated earlier that the mathematical solution for the torsion of certain solid and thin-walled sections is complex and beyond the scope of this text. In such cases it is extremely fortunate that an analogy exists known as the *membrane analogy*, which provides a very convenient mental picture of the way in which stresses build up in such components and allows experimental determination of their values.

It can be shown that the mathematical solution for elastic torsion problems involving partial differential equations is identical in form to that for a thin membrane lightly stretched over a hole. The membrane normally used for visualisation is a soap film. Provided that the hole used is the same shape as the cross-section of the shaft in question and that air pressure is maintained on one side of the membrane, the following relationships exist:

(a) the torque carried by the section is equal to twice the volume enclosed by the membrane;
(b) the shear stress at any point in the section is proportional to the slope of the membrane at that point (Fig. 5.9);
(c) the direction of the shear stress at any point in the section is always at right angles to the slope of the membrane at the same point.

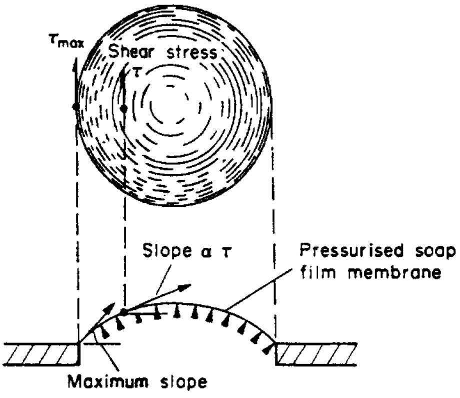


Fig. 5.9. Membrane analog.

Application of the above rules to the open sections of Fig. 5.2 shows that each section will carry approximately the same torque at the same maximum shear stress since the volumes enclosed by the membranes and the maximum slopes of the membranes are approximately equal in each case.

The membrane analogy is particularly powerful in the study of the comparative torsional properties of different sections without the need for detailed calculations. For example, it should be evident from the volume relationship (a) above that if two cross-sections have the same area, that which is nearer to circular will be the stronger in torsion since it will produce the greatest enclosed volume.

The analogy also helps to support the theory used for thin-walled open sections in §5.3 when thin rectangular sections are taken to have the same torsional stiffness be they left as a single rectangle or bent into open tubes, angle sections, channel sections, etc.

From the slope relationship (b) the greatest shear stresses usually occur at the boundary of the thickest parts of the section. They are usually high at positions where the boundary is sharply concave but low at the ends of outstanding flanges.

5.11. Effect of warping of open sections

In the preceding paragraphs it has been assumed that the torque is applied at the ends of the member and that all sections are free to warp. In practice, however, there are often cases where one or more sections of a member are constrained in some way so that cross-sections remain plane, i.e. warping is prevented. Whilst this has little effect on the angle of twist of certain solid cross-sections, e.g. rectangular or elliptical sections where the length is significantly greater than the section dimensions, it may have a considerable effect on the twist of open sections. In the latter case the constraint of warping is often accompanied by considerable bending of the flanges. Detailed treatment of warping is beyond the scope of this text† and it is sufficient to note here that when warping is restrained, angles of twist are generally reduced and hence torsional stiffnesses increased.

† S. Timoshenko and J.N. Goodier, *Theory of Elasticity*, McGraw-Hill, New York.

Examples

Example 5.1

A rectangular steel bar 25 mm wide and 38 mm deep is subjected to a torque of 450 Nm. Estimate the maximum shear stress set up in the material of the bar and the angle of twist, using the experimentally derived formulate stated in §5.1.

What percentage error would be involved in each case if the approximate equations are used?

For steel, take $G = 80$ GN/m^2.

Solution

The maximum shear stress is given by eqn. (5.1):

$$\tau_{max} = \frac{T}{k_1 d b^2}$$

In this case $d = 38$ mm, $b = 25$ mm, i.e. $d/b = 1.52$ and k_1 for d/b of $1.5 = 0.231$.

$$\therefore \qquad \tau_{max} = \frac{450}{0.231 \times 38 \times 10^{-3} \times (25 \times 10^{-3})^2} = \mathbf{82\ MN/m^2}$$

The angle of twist per unit length is given by eqn. (5.2):

$$\frac{\theta}{L} = \frac{T}{k_2 d b^3 G}$$

and from the tables, for $d/b = 1.5$, k_2 is 0.196.

$$\therefore \qquad \theta = \frac{450}{0.196 \times 38 \times 10^{-3} \times (25 \times 10^{-3})^3 \times 80 \times 10^9}$$

$$= 0.0483 \text{ rad/m}$$

$$= \mathbf{2.77\ degrees/m}$$

Approximately

$$\tau_{max} = \frac{T}{db^2}(3 + 1.8b/d)$$

$$= \frac{450}{38 \times 10^{-3} \times (25 \times 10^{-3})^2}\left(3 + 1.8 \times \frac{25}{38}\right)$$

$$= \frac{450}{2.375 \times 10^{-5}}(3 + 1.184) = \mathbf{79.3\ MN/m^2}$$

Therefore percentage error

$$= \left(\frac{79.3 - 82.02}{82.02}\right) 100 = \mathbf{-3.3\%}$$

Again, approximately,

$$\theta = \frac{42TJ}{GA^4} \text{ per metre}$$

Now $$J = I_{xx} + I_{yy} = \frac{bd^3}{12} + \frac{db^3}{12} = \frac{bd}{12}(d^2 + b^2)$$

$$= \frac{25 \times 38(25^2 + 38^2)}{12 \times 10^{12}} = 0.1638 \times 10^{-6}\ \text{m}^4$$

$\therefore$ $$\theta = \frac{42 \times 450 \times 0.164 \times 10^{-6}}{80 \times 10^9 \times (25 \times 38 \times 10^{-6})^4} = 0.0476\ \text{rad/m}$$

$$= 2.73\ \text{degrees/m}$$

$$\text{Percentage error} = \left(\frac{2.73 - 2.77}{2.77}\right) 100 = \mathbf{-1.44\%}$$

Example 5.2

Compare the torsional stiffness of the following cross-sections which can be assumed to be of unit length. Compare also the maximum shear stresses set up in each case:

(a) a hollow tube 40 mm mean diameter and 2 mm wall thickness;
(b) the same tube with a 2 mm wide saw-cut along its length;
(c) a rectangular solid bar, side ratio 4 to 1, having the same cross-sectional area as that enclosed by the mean diameter of the hollow tube;
(d) an equal-leg angle section having the same perimeter and thickness as the tube;
(e) a square box section having the same perimeter and thickness as the tube.

Solution

(a) In the case of the *closed hollow tube* we can apply the standard torsion equation

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$$

together with the simplified formula for the polar moment of area J of thin tubes,

$$J = 2\pi r^3 t$$

$\therefore$ $$\text{torsional stiffness} = \frac{T}{\theta} = \frac{GJ}{L} = \frac{2\pi \times (20 \times 10^{-3})^3 \times 2 \times 10^{-3} G}{1}$$

$$= 100.5 \times 10^{-9} G$$

$$\text{maximum shear stress} = \frac{TR}{J} = \frac{20 \times 10^{-3} \times T}{2\pi \times (20 \times 10^{-3})^3 \times 2 \times 10^{-3}}$$

$$= \mathbf{0.198 \times 10^6 T}$$

(b) *Tube with split*
From the work of §5.4,

$$\text{angle of twist/unit length} = \frac{\theta}{L} = \frac{T}{k_2\, db^3 G} = \frac{T}{k_2 (2\pi r - x) t^3 G}$$

$$\therefore \qquad \text{torsional stiffness} = \frac{T}{\theta} = \frac{k_2(2\pi r - x)t^3 G}{L}$$

$$= \frac{0.333[2\pi \times 20 \times 10^{-3} - 2 \times 10^{-3}](2 \times 10^{-3})^3 G}{1}$$

$$= 0.333(125.8 - 2)8 \times 10^{-12} G$$

$$= \mathbf{329.8 \times 10^{-12} G}$$

$$\text{Maximum shear stress} = \frac{T}{k_1 d b^2}$$

$$= \frac{T}{0.333 \times 123.8 \times 10^{-3} \times (2 \times 10^{-8})^2}$$

$$= \mathbf{6.06 \times 10^6 T}$$

i.e. splitting the tube along its length has reduced the stiffness by a factor of approximately **300**, the maximum stress increasing by approximately **30** times.

(c) *Rectangular bar*

$$\text{Area of hollow tube} = \text{area of bar}$$

$$= \pi \times (20 \times 10^{-3})^2$$

$$\therefore \qquad 4b^2 = 8\pi \times 10^{-4}$$

$$b^2 = 2\pi \times 10^{-4}$$

$$\therefore \qquad b = 2.5 \times 10^{-2}\ \text{m} = 25\ \text{mm}$$

$$\therefore \qquad d = 4b = 100\ \text{mm}$$

d/b ratio $= 4$

$$\therefore \qquad k_1 = 0.282 \quad \text{and} \quad k_2 = 0.281$$

Therefore from eqn. (5.2),

$$\frac{\theta}{L} = \frac{T}{k_2 d b^3 G}$$

$$\therefore \qquad \frac{T}{\theta} = 0.281 \times 10 \times 10^{-2} \times (2.5 \times 10^{-2})^3 G$$

$$= 43.9 \times 10^{-8} G$$

$$= \mathbf{439 \times 10^{-9} G}$$

From eqn. (5.1),

$$\tau_{max} = \frac{T}{k_1 d b^2} = \frac{T}{0.282 \times 10 \times 10^{-2} \times (2.5 \times 10^{-2})^2}$$

$$= \mathbf{0.057 \times 10^6 T}$$

(d) *Equal-leg angle section*

$$\text{Perimeter of angle} = \text{perimeter of tube}$$
$$= 2\pi \times 20 \times 10^{-3}\ \text{m}$$
$$\therefore \quad \text{Length of side } d = 20\pi \times 10^{-3}\ \text{m}$$

Therefore applying eqn. (5.12),

$$\frac{\theta}{L} = \frac{3T}{G\Sigma db^3}$$
$$= \frac{3T}{2G \times 20\pi \times 10^{-3} \times (2 \times 10^{-3})^3}$$
$$\therefore \quad \frac{T}{\theta} = (2G \times 20\pi \times 8 \times 10^{-12})/3$$
$$= \mathbf{0.335 \times 10^{-9}} \boldsymbol{G}$$

And from eqn. (5.11)

$$\tau_{max} = \frac{3T}{\Sigma db^2}$$
$$= \frac{3T}{2 \times 20\pi \times 10^{-3} \times (2 \times 10^{-3})^2}$$
$$= \mathbf{5.97 \times 10^6}\ \boldsymbol{T}$$

(e) *Square box section (closed)*

$$\text{Perimeter } s = \text{tube perimeter} = 2\pi \times 20 \times 10^{-3}\ \text{m}$$
$$\therefore \quad \text{side length} = \frac{2\pi \times 20 \times 10^{-3}}{4} = \pi \times 10^{-2}\ \text{m}$$

Therefore area enclosed by median line

$$= A = (\pi \times 10^{-2})^2$$

From eqn. (5.16),

$$\theta = \frac{TLs}{4A^2Gt}$$
$$\therefore \quad \frac{T}{\theta} = \frac{4 \times (\pi \times 10^{-2})^4 G \times 2 \times 10^{-3}}{1 \times 2\pi \times 20 \times 10^{-3}}$$
$$= \mathbf{62 \times 10^{-9}} \boldsymbol{G}$$

From eqn. (5.15)

$$\tau_{max} = \frac{T}{2At} = \frac{T}{2 \times (\pi \times 10^{-2})^2 \times 2 \times 10^{-3}}$$
$$= \mathbf{0.253 \times 10^6} \boldsymbol{T}$$

Example 5.3

A thin-walled member 1.2 m long has the cross-section shown in Fig. 5.10. Determine the maximum torque which can be carried by the section if the angle of twist is limited to 10°. What will be the maximum shear stress when this maximum torque is applied? For the material of the member $G = 80$ GN/m^2.

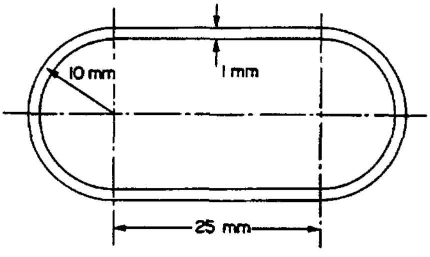


Fig. 5.10.

Solution

This problem is of the type considered in §5.6, a solution depending upon the length of, and the area enclosed by, the median line.

Now, perimeter of median line $= s = (2 \times 25 + 2\pi \times 10)$ mm

$$= 112.8 \text{ mm}$$

area enclosed by median $= A = (20 \times 25 + \pi \times 10^2)$ mm^2

$$= 814.2 \text{ mm}^2$$

From eqn (5.16),
$$\theta = \frac{TLs}{4A^2\,Gt}$$

$$\therefore \quad \frac{10 \times 2\pi}{360} = \frac{T \times 1.2 \times 112.8 \times 10^{-3}}{4(814.2 \times 10^{-6})^2 \times 80 \times 10^9 \times 1 \times 10^{-3}}$$

i.e. maximum torque possible,

$$T = \frac{20\pi \times 4 \times 814.2^2 \times 80 \times 10^{-6}}{360 \times 1.2 \times 112.8 \times 10^{-3}}$$

$$= \mathbf{273\ Nm}$$

From eqn. (5.15),
$$\tau_{\max} = \frac{T}{2At}$$

$$= \frac{273}{2 \times 814.2 \times 10^{-6} \times 1 \times 10^{-3}}$$

$$= 168 \times 10^6 = \mathbf{168\ MN/m^2}$$

The maximum stress produced is 168 MN/m^2.

Example 5.4

The median dimensions of the two cells shown in the cellular section of Fig. 5.6 are $A_1 =$ 20 mm $\times$ 40 mm and $A_2 =$ 50 mm $\times$ 40 mm with wall thicknesses $t_1 = 2$ mm, $t_2 = 1.5$ mm and $t_3 = 3$ mm. If the section is subjected to a torque of 320 Nm, determine the angle of twist per unit length and the maximum shear stress set up. The section is constructed from a light alloy with a modulus of rigidity $G = 30$ GN/m^2.

Solution

From eqn. (5.18), $$320 = 2(\tau_1 \times 2 \times 20 \times 40 + \tau_2 \times 1.5 \times 50 \times 40)10^{-9} \qquad (1)$$

From eqn. (5.19),

$$2 \times 30 \times 10^9 \times \theta = \frac{1}{20 \times 40 \times 10^{-6}}[\tau_1(40 + 2 \times 20)10^{-3} + \tau_3 \times 40 \times 10^{-3}] \qquad (2)$$

and $$2 \times 30 \times 10^9 \times \theta = \frac{1}{50 \times 40 \times 10^{-6}}[\tau_2(40 + 2 \times 50)10^{-3} - \tau_3 \times 40 \times 10^{-3}] \qquad (3)$$

Equating (2) and (3),

From eqn. (5.17), $$2\tau_1 = 1.5\tau_2 + 3\tau_3 \qquad (4)$$

$$\tfrac{1}{8}[80\tau_1 + 40\tau_3] = \tfrac{1}{20}[140\tau_2 - 40\tau_3]$$

Multiply through by 40,

$$400\tau_1 + 200\tau_3 = 280\tau_2 - 80\tau_3$$

$$40\tau_1 = 28\tau_2 - 28\tau_3 \qquad (5)$$

(5) × 60/28 $$85.7\tau_1 = 60\tau_2 - 60\tau_3 \qquad (6)$$

But, from (4), multiplied by 20,

$$40\tau_1 = 30\tau_2 + 60\tau_3 \qquad (7)$$

(6) + (7), $$125.7\tau_1 = 90\tau_2 \qquad (8)$$

and from (1), $$320 = (3.2\tau_1 + 6\tau_2)10^{-6}$$

$$320 \times 10^6 = 3.2\tau_1 + 6\tau_2 \qquad (9)$$

substituting for τ_2 from (8),

$$320 \times 10^6 = 3.2\tau_1 + 6 \times \frac{125.7}{90}\tau_1$$

$$= 3.2\tau_1 + 8.4\tau_1$$

$$\therefore \qquad \tau_1 = \frac{320 \times 10^6}{11.6} = 27.6 \times 10^6 = 27.6 \text{ MN/m}^2$$

From (8),

$$\tau_2 = \frac{125.7}{90} \times 27.6 = 38.6 \text{ MN/m}^2$$

From (4),

$$\tau_3 = \tfrac{1}{3}(2 \times 27.6 - 1.5 \times 38.6)$$

$$= \tfrac{1}{3}(55.2 - 57.9) = \tfrac{1}{3} \times (-2.7) = -0.9 \text{ MN/m}^2$$

The negative sign indicates that the direction of shear flow in the wall of thickness t_3 is reversed from that shown in Fig. 5.6.

The maximum shear stress present in the section is thus **38.6 MN/m²** in the 1.5 mm wall thickness.

From (3),

$$2 \times 30 \times 10^9 \times \theta = \frac{(140t_2 - 40t_3)}{50 \times 40 \times 10^{-3}}$$

$$= \frac{140 \times 38.6 \times 10^6 - 40(-0.9 \times 10^6)}{50 \times 40 \times 10^{-3} \times 2 \times 30 \times 10^9}$$

$$= \frac{(5.40 + 0.036)}{120} \text{ radian}$$

$$= \frac{5.440}{120} \times \frac{360}{2\pi} = \mathbf{2.6^\circ}$$

The angle of twist of the section is 2.6°.

Problems

5.1 (A). A 40 mm × 20 mm rectangular steel shaft is subjected to a torque of 1 kNm. What will be the approximate position and magnitude of the maximum shear stress set up in the shaft? Determine also the corresponding angle of twist per metre length of the shaft.

For the bar material $G = 80$ GN/m². [254 MN/m²; 9.78°/m.]

5.2 (B). An extruded light alloy angle section has dimensions 80 mm × 60 mm × 4 mm and is subjected to a torque of 20 Nm. If $G = 30$ GN/m² determine the maximum shear stress and the angle of twist per unit length. How would the former answer change if one considered the stress concentration effect at the fillet owing to a fillet radius of 10 mm? [27.6 MN/m²; 13.2°/m; 30.4 MN/m².]

5.3 (B). Compare the torsional rigidities of the following sections:

(a) a hollow tube 30 mm outside diameter and 1.5 mm thick; [2.7×10^{-8} G.]
(b) the same tube split along its length with a 1 mm gap; [0.0996×10^{-9} G.]
(c) an equal leg angle section having the same perimeter and thickness as (b); [0.0996×10^{-9} G.]
(d) a square box section with side length 30 mm and 1.5 mm wall thickness; [3.48×10^{-8} G.]
(e) a rectangular solid bar, side ratio 2.5 to 1, having the same metal cross-sectional area as the hollow tube. [1.79×10^{-9} G.]

Compare also the maximum stresses arising in each case.
[$0.522 \times 10^6 T$; $15 \times 10^6 T$; $15 \times 10^6 T$; $0.41 \times 10^6 T$; $4.05 \times 10^6 T$.]

5.4 (B). The spring return of an interlocking device for a cold room door is to be made of a rectangular strip of spring steel loaded in torsion. The width of the strip cannot be greater than 10 mm and the effective length 100 mm. Calculate the thickness of the strip if the torque is to be 15 Nm at an angle of 10° and if the torsion yield stress of 420 MN/m² is not to be exceeded at this angle. Take G as 83 GN/m².

Assume $k_1 = k_2 = \frac{1}{3}$. [3.27 mm.]

5.5 (B). A thin-walled member of 2 m long has the section shown in Fig. 5.11. Determine the torque that can be applied and the angle of twist achieved if the maximum shear stress is limited to 30 MN/m². $G = 250$ GN/m². [42.85 Nm; 0.99°.]

5.6 (B). A steel sheet, 400 mm wide by 2 mm thick, is to be formed into a hollow section by bending through 360° and butt-welding the long edges together. The shape may be (a) circular, (b) square, (c) a rectangle 140 mm × 60 mm. Assume a median length of 400 mm in each case (i.e. no stretching) and square corners for non-circular sections. The allowable shearing stress is 90 MN/m². For each of the shapes listed determine the magnitude of the maximum permissible torque and the angles of twist per metre length if $G = 80$ GN/m². [4.58, 3.6, 3.01 kNm; 1°, 1°17′, 1°31′.]

5.7 (B). Figure 5.12 represents the cross-section of an aircraft fuselage made of aluminium alloy. The sheet thicknesses are: 1 mm from A to B and C to D; 0.8 mm from B to C and 0.7 mm from D to A. For a maximum torque of 5000 Nm determine the magnitude of the maximum shear stress and the angle of twist/metre length. $G = 30$ GN/m². [50 MN/m²; 0.0097 rad.]

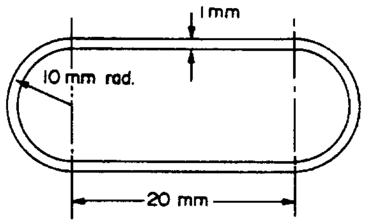


Fig. 5.11.

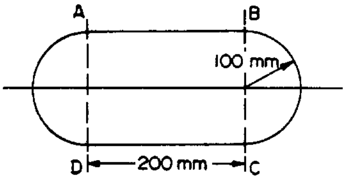


Fig. 5.12.

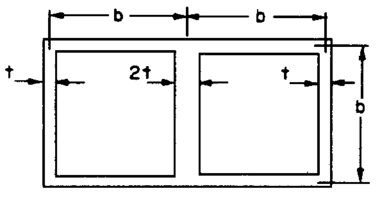


Fig. 5.13.

5.8 (B/C). Show that for the symmetrical section shown in Fig. 5.13 there is no stress in the central web. Show also that the shear stress in the remainder of the section has a value of $T/4tb^2$.

5.9 (C). A washing machine agitator of the cross-section shown in Fig. 5.14 acts as a torsional member subjected to a torque T. The central tube is 100 mm internal diameter and 12 mm thick; the rectangular bars are 50 mm × 18 mm section. Assuming that the total torque carried by the member is given by

$$T = T_{\text{tube}} + 4T_{\text{bar}}$$

determine the maximum value of T which the shaft can carry if the maximum stress is limited to 80 MN/m^2.
(Hint: equate angles of twist of tube and bar.) [19.1 kNm.]

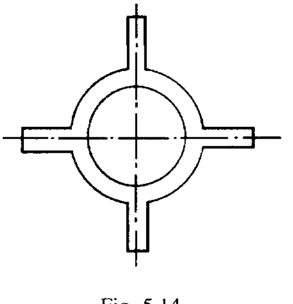


Fig. 5.14.

5.10 (C). The cross-section of an aeroplane elevator is shown in Fig. 5.15. If the elevator is 2 m long and constructed from aluminium alloy with $G = 30$ GN/m^2, calculate the total angle of twist of the section and the magnitude of the shear stress in each part for an applied torque of 40 Nm.

[0.0169°; 3.43, 2.58, 1.15 × 10^5 N/m^2.]

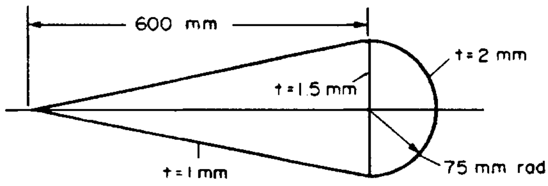


Fig. 5.15.

5.11 (B/C). Develop a relationship between torque and angle of twist for a closed uniform tube of thin-walled non-circular section and use this to derive the twist per unit length for a strip of thin rectangular cross-section.

Use the above relationship to show that, for the same torque, the ratio of angular twist per until length for a closed square-section tube to that for the same section but opened by a longitudinal slit and free to warp is approximately $4t^2/3b^2$, where t, the material thickness, is much less than the mean width b of the cross-section.

[C.E.I.]

5.12 (C). A torsional member used for stirring a chemical process is made of a circular tube to which is welded four rectangular strips as shown in Fig. 5.16. The tube has inner and outer diameters of 94 mm and 100 mm respectively, each strip is 50 mm × 18 mm, and the stirrer is 3 m in length.

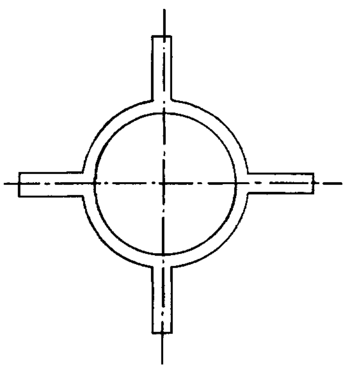


Fig. 5.16.

If the maximum shearing stress in any part of the cross-section is limited to 56 MN/m^2, neglecting any stress concentration, calculate the maximum torque which can be carried by the stirrer and the resulting angle of twist over the full length.

For torsion of rectangular sections the torque T is related to the maximum shearing stress, τ_{max}, and angle of twist, θ, in radians per unit length, as follows:

$$T = k_1 b d^2 \tau_{max} = k_2 b d^3 \, G\theta$$

where b is the longer and d the shorter side of the rectangle and in this case, $k_1 = 0.264$, $k_2 = 0.258$ and $G = 83$ GN/m^2.

[C.E.I.] [2.83 kNm, 2.4°.]

5.13 (C). A long tube is subjected to a torque of 200 Nm. The tube has the double-cell, thin-walled, effective cross-section illustrated in Fig. 5.17. Assuming that no buckling occurs and that the twist per unit length of the tube is constant, determine the maximum shear stresses in each wall of the tube.

[C.E.I.] [0.76, 1.01, 0.19 MN/m^2.]

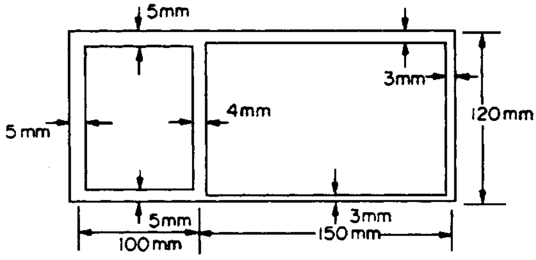


Fig. 5.17.

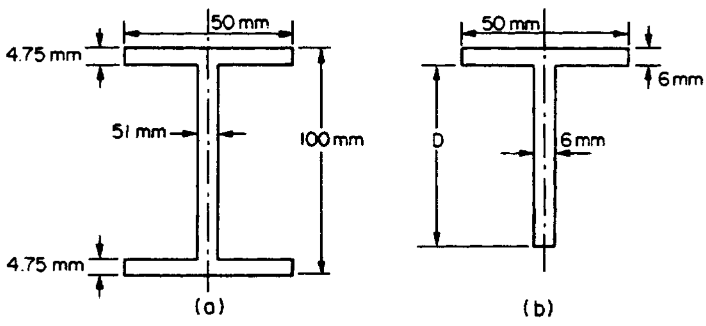


Fig. 5.18.

5.14 (B/C). An I-section has the dimensions shown in Fig. 5.18(a), and is subjected to an axial torque. Find the maximum value of the torque if the shear stress in the material is limited to 56 MN/m^2 and the twist per metre length is limited to 9°. Assume the modulus of rigidity G for the material is 82 GN/m^2.

If the I-section is replaced by a T-section made of the same material and transmits the same torque, what will be the limb length, D, of the T-section and the angle of twist per metre length? Assume the T-section is subjected to the same limiting conditioning as the I-section and that it has the dimensions shown in Fig. 5.18(b). For narrow rectangular sections assume k values of $\frac{1}{3}$ in the formulae for torque and angle of twist.

[B.P.] [0.081 m; 6.5°/m.]

5.15 (B/C). (a) An aluminium sheet, 600 mm wide and 4 mm thick, is to be formed into a hollow section tube by bending through 360° and butt-welding the long edges together. The cross-section shape may be either circular or square.

Assuming a median length of 600 mm in each case, i.e. assuming no stretching occurs, determine the maximum torque that can be carried and the resulting angle of twist per metre length in each case.

Maximum allowable shearing stress = 65 MN/m^2, shear modulus G = 40 GN/m^2.

(b) What would be the effect on the stiffness per metre length of each type of section of a narrow saw-cut through the tube wall along the length of the tube? In the case of the square section assume that the cut is taken along the centre of one face.

[B.P.] [14.9 kNm, 0.975°; 11.7k Nm, 1.24°; reduction 1690 times, reduction 1050 times.]

5.16 (B). The two sections shown in Fig. 5.19 are under consideration for an engineering application which includes both bending and applied torque. Make a critical comparison of the strengths of the two sections under the two modes of loading and make a recommendation as to the section which should be adopted. The material to be used is to be the same for both sections.

The rectangular section torsion constants k_1 and k_2 may be found in terms of the section d/b ratio from Table 3.1.

[Tubular]

5.17 (B). Compare the angles of twist of the following sections when each is subjected to the same torque of 3 kNm;

(a) circular tube, 80 mm outside diameter, 6 mm thick;
(b) square tube, 52 mm side length (median dimension), 6 mm thick;

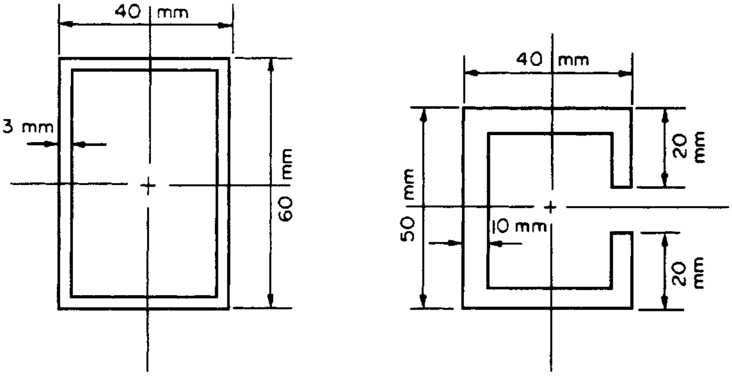


Fig. 5.19.

(c) circular tube as (a) but with additional four rectangular fins 80 mm long by 15 mm wide symmetrically placed around the tube periphery.

All sections have the same length of 2 m and $G = 80$ GN/m^2 [0.039 rad; 0.088 rad; 0.038 rad]
To what maximum torque can sections (a), (b) and (c) be subjected if the maximum shear stress is limited to 100 MN/m^2? [4.8 kNm; 3.24 kNm; 5.7 kNm]

What maximum angle of twist can be accepted by tube (c) for the same limiting shear stress? [0.0625 rad]

5.18 (B). Figure 5.20 shows part of the stirring mechanism for a chemical process, consisting of a circular stainless-steel tube of length 2 m, outside diameter 75 mm and wall thickness 6 mm, welded onto a square mild-steel tube of length 1.5 m. Four blades of rectangular section stainless-steel, 100 mm × 15 mm, are welded along the full length of the stainless-steel tube as shown.

(a) Select a suitable section for the square tube from the available stock list below so that when the maximum allowable shear stress of 58 MN/m^2 is reached in the stainless-steel, the shear stress in the mild steel of the square tube does not exceed 130 MN/m^2.

Section	Dimension	Wall thickness	Torsion constant (J equiv)
1	50 × 50 mm	5 mm	476000 mm^4
2	60 × 60 mm	4 mm	724000 mm^4
3	70 × 70 mm	3.6 mm	1080000 mm^4.

(b) Having selected an appropriate mild steel tube, determine how much the entire mechanism will twist during operation at a constant torque of 3 kNm.

The shear modulus of stainless steel is 78 GN/m^2 and of mild steel is 83 GN/m^2. Neglect the effect of any stress concentration. [50 mm × 50 mm; 0.152 rad]

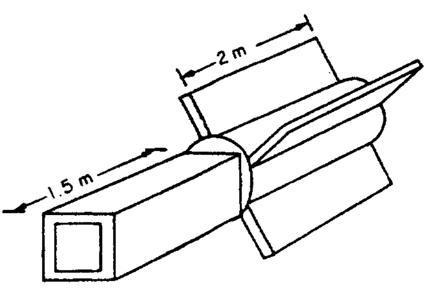


Fig. 5.20.

5.19 (B). Figure 5.21 shows the cross-section of a thin-walled fabricated service conduit used for the protection of long runs of electrical wiring in a production plant. The lower plate AB may be removed for inspection and re-cabling purposes.

Owing to the method by which the conduit is supported and the weight of pipes/wires that it carries, the section is subjected to a torque of 130 Nm. With plate AB assumed in position, determine the maximum shear stress set up in the walls of the conduit. What will be the angle of twist per unit length?

By consideration of maximum stress levels and angles of twist, establish whether the section design is appropriate for the removal of plate AB for maintenance purposes assuming that the same torque remains applied. If modifications are deemed to be necessary suggest suitable measures.

For the conduit material $G = 80\ \text{GN/m}^2$ and maximum allowable shear stress $= 180\ \text{MN/m}^2$.

[167 MN/m^2; 39°/m]

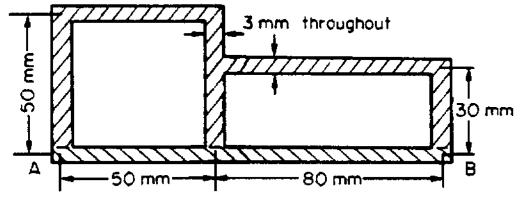


Fig. 5.21.

5.20 (B). (a) Figure 5.22 shows the cross-section of a thin-walled duct which forms part of a fluid transfer system. The wire mesh, FC, through which sediment is allowed to pass, may be assumed to contribute no strength

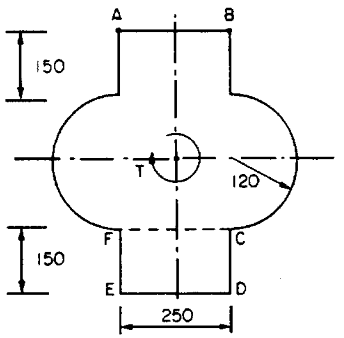


Fig. 5.22. All dimensions (mm) may be taken as median dimensions.

to the section. Owing to the method of support, the weight of the fluid and duct introduces a torque to the section which may be assumed uniform.

If the maximum shear stress in the duct material is limited to $150\ \text{MN/m}^2$; determine the maximum torque which can be tolerated and the angle of twist per metre length when this maximum torque is applied. For the duct material $G = 85\ \text{GN/m}^2$. [432.6 kNm; 0.516°/m]

(b) In order to facilitate cleaning and inspection of the duct, plates AB and ED are removable. What would be the effect on the results of part (a) if plate AB were inadvertently left off over part of the duct length after inspection? [5.12 kNm; 12.6°/m]

5.21 (C). Figure 5.8 shows a polymer extrusion of wall thickness 4 mm. The section is to be stiffened by the insertion of an aluminium I section as shown, the centre web of the polymer extrusion having been removed. The I section wall thickness is also 4 mm.

If $G = 3.3\ \text{GN/m}^2$ for the polymer and $70\ \text{GN/m}^2$ for the aluminium, what increase in stiffness is achieved? What increase in torque is allowable, if the design is governed by maximum allowable stresses of $5\ \text{MN/m}^2$ and $100\ \text{MN/m}^2$ in the polymer and aluminium respectively? [258%, 7.4%]