

Dynamic vibration absorber

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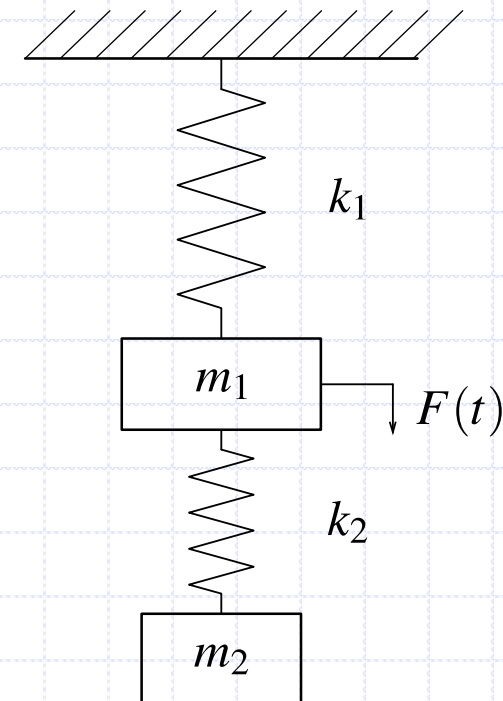
Iva Petříková

Dynamic vibration absorber

Figure represents a simple two mass system with harmonic force $F_0 \sin \omega t$ applied to m_1 .

In order to avoid undesirable resonance conditions in many applications, the system stiffness and inertia characteristics must be changed. Another approach, to alleviate the resonant conditions, is to convert the single degree of freedom system (DOF) to a two DOF by adding an auxiliary spring and mass system.

The parameters of the added system can be selected in such a manner that the vibration of the main mass is eliminated.



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- ◆ System with 2 DOF undamped

$$m_1 \ddot{x}_1 = k_2 (x_2 - x_1) - k_1 x_1 + F_0 \sin \omega t \quad (1)$$

$$m_2 \ddot{x}_2 = -k_2 (x_2 - x_1) \quad (2)$$

- ◆ Supposed solutions,

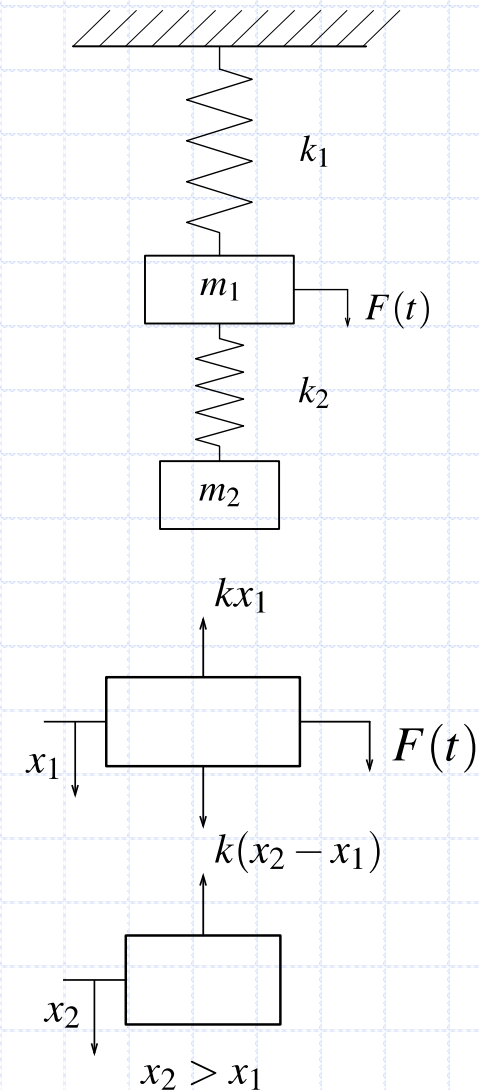
$$x_1 = X_1 \sin \omega t \quad \ddot{x}_1 = -\omega^2 X_1 \sin \omega t$$

$$x_2 = X_2 \sin \omega t \quad \ddot{x}_2 = -\omega^2 X_2 \sin \omega t$$

- ◆ Substitution to (1) a (2)

$$(k_1 + k_2 - \omega^2 m_1) X_1 - k_2 X_2 = F_0 \quad (3)$$

$$-k_2 X_1 + (k_2 - \omega^2 m_1) X_2 = 0 \quad (4)$$



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- ◆ Eq. (1) is divided by k_1 and eq. (2) k_2 denote relations

$$\sqrt{\frac{k_1}{m_1}} = \omega_{11} \quad \text{natural frequency of single DOF with mass } m_1$$

$$\sqrt{\frac{k_2}{m_2}} = \omega_{22} \quad \text{natural frequency of single DOF with mass } m_2$$

$$\frac{F_0}{k_1} = X_{ST} \quad \text{static deflection of mass 1}$$

$$\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_{11}} \right)^2 \right] X_1 - \frac{k_2}{k_1} X_2 = \frac{F_0}{k_1}$$

$$-X_1 + \left[1 - \left(\frac{\omega}{\omega_{22}} \right)^2 \right] X_2 = 0$$

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- ◆ Amplitude of two masses are obtained from solution of equations (3) and (4)

$$\frac{X_1}{X_{ST}} = \frac{1 - \left(\frac{\omega}{\omega_{22}}\right)^2}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_{11}}\right)^2\right] \left[1 - \left(\frac{\omega}{\omega_{22}}\right)^2\right] - \frac{k_2}{k_1}} \quad (5)$$

$$\frac{X_2}{X_{ST}} = \frac{1}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_{11}}\right)^2\right] \left[1 - \left(\frac{\omega}{\omega_{22}}\right)^2\right] - \frac{k_2}{k_1}} \quad (6)$$

- ◆ It is evident from these equations that the amplitude X_1 of the main system becomes zero when the exciting frequency ω coincides with the natural frequency ω_{22} of mass 2. For this frequency amplitude X_2 is equal to:

$$X_2 = -\frac{k_1}{k_2} X_{ST} = -\frac{F_0}{k_2} \quad (7)$$

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- ◆ The negative sign indicates that amplitude X_2 is out of phase with exciting force. If $X_1=0$, the force $k_2 X_2$ exerted by spring 2 on mass m_1 is equal and opposite to the impressed force F_0 . For exciting frequency of the system yields

$$\omega = \omega_{22} = \sqrt{\frac{k_2}{m_2}}$$

$$k_2 X_2 = \omega^2 m_2 X_2 = -F_0$$

- ◆ Amplitude of main mass can reach two resonance frequency.

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Resonant frequency can be obtained by setting the denominator of the amplitude equation (5) and (6) to zero.

$$\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_{11}}\right)^2\right] \left[1 - \left(\frac{\omega}{\omega_{22}}\right)^2\right] - \frac{k_2}{k_1} = 0$$

